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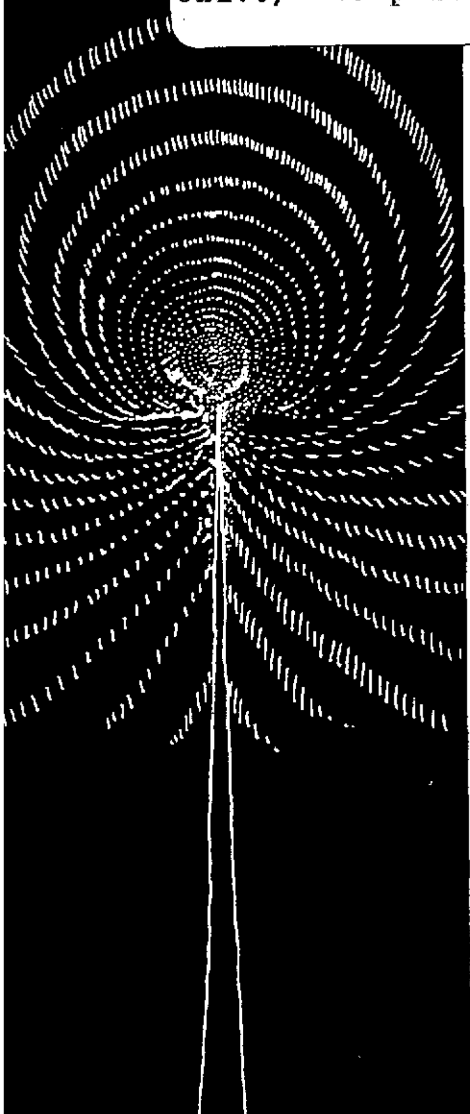

By

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A NOTE ON THE EFFECT OF BENDING STIFFNESS OF STRINGERS ATTACHED TO A PLATE WITH A CRACK

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ABSTRACT

An infinite stringer which is assumed to be partially bonded to a plate through a layer of adhesive is considered. The stringer is assumed to have bending as well as longitudinal stiffness. The effect of the stringer's bending rigidity on the stress intensity factor at the tip of the crack is illustrated. Shear stress distribution between the plate and the stringer and the stress intensity factors will be obtained from the solution of a system of Fredholm integral equations which represent the continuity of displacements along the line of bond.

INTRODUCTION

In a previous paper [1], the effect of a partially debonded infinite stringer on the stress intensity factor at the crack tip has been investigated. It was assumed that the stringer which has been located perpendicular to the crack had no bending stiffness. The conclusion was that the stiffening effect of the stringer was very small if it is not placed very close to the crack tip or on the crack itself and/or if the length of the debonding is approximately more than twice the crack length (see [1]).

The importance of the problem is mainly due to the use of stringers in structures to prevent catastrophic failure. Usually the stringers used will have some bending stiffness in the plane of the plate which may not be negligible. Hence the purpose of the present paper is to study its effect on fracture arrest.

The method used here will be similar to the one used in [1] and [2]. Again, this approach does not put any restriction on the number of stringers.

FORMULATION OF THE PROBLEM

The perturbation problem, where the crack surfaces are subject to uniform pressure q and the loading at infinity is zero, will be considered. Adhesive will be treated as a shear spring and the shear stresses transmitted through the adhesive will be considered as body forces for the plate solution (see [1]).

If L denotes the line $x = d$, $b \leq y < \infty$, in view of the symmetry, the continuity of the displacements can be written as (see Figure 1)

$$[u_p(y) + iv_p(y)] - [u_s(y) + iv_s(y)] = \frac{h_a}{d_s \mu_a} [P_1(y) + iP_2(y)], \quad y \text{ on } L \tag{1}$$

where as in [1], μ_a is the shear modulus of the adhesive, d_s is the width of the stringer and h_a is the thickness of the adhesive. $P_1(y)$ and $P_2(y)$ are the horizontal and the vertical components of the shear force per unit length of the stringer (see Figure 2).

Here, the displacements of the plate $u_p + iv_p$ and of the stringer $u_s + iv_s$ can be given as follows:

$$u_s(y) = \frac{1}{6E_s I_s} \int_L k_{s1}(y, y_o) P_1(y_o) dy_o$$

$$v_s(y) = \frac{1}{A_s E_s} \int_L k_{s2}(y, y_o) P_2(y_o) dy_o, \quad y \text{ on } L \tag{2}$$

where

$$k_{s1}(y, y_o) = \begin{cases} y_o^3 + 3y^2 y_o, & y_o > y \\ y^3 + 3yy_o^2, & y_o < y \end{cases}$$

$$k_{s2}(y, y_o) = \begin{cases} y, & y_o > y \\ y_o, & y_o < y \end{cases} \tag{3}$$

$$v_s(0) = 0$$

and E_s, A_s and I_s are the elastic modulus, cross-sectional area and the moment of inertia of the stringer respectively.

On the other hand

$$u_p(y) + iv_p(y) = qk_o(y) + \int_L [k_{p1}(y,y_o)S(y_o) + k_{p2}(y,y_o)\overline{S(y_o)}] dy_o , \qquad y \text{ on } L \tag{4}$$

where as in [1] and [2]

$$k_o(y) = \frac{1}{4\mu_p} [\kappa\sqrt{z^2-a^2} - \sqrt{\bar{z}^2-a^2} + \frac{(\bar{z}-z)\bar{z}}{\sqrt{\bar{z}^2-a^2}} + (1-\kappa)z]$$

$$= \frac{1}{4\mu_p} [(\kappa-1)\mathrm{Re}\sqrt{z^2-a^2} + (1-\kappa)x - 2y\mathrm{Im}\frac{z}{\sqrt{z^2-a^2}}]$$

$$+ \frac{i}{4\mu_p} [(\kappa+1)\mathrm{Im}\sqrt{z^2-a^2} + (1-\kappa)y - 2y\mathrm{Re}\frac{z}{\sqrt{z^2-a^2}}]$$

$$= \frac{1}{4\mu_p} [f_1(y) + if_2(y)] , \qquad f_2(0) = 0, \ |x| > a \tag{5}$$

where $\kappa = (3-\nu)/(1+\nu)$, μ_p, ν are the elastic constants of the plate, 2a is the crack length, d is the distance of the stringer to the mid-point of the crack, and

$$z = d + iy , \qquad y \text{ on } L$$

$$S(y_o) = \frac{P_1(y_o) + iP_2(y_o)}{2\pi h_p(1+\kappa)} \tag{6}$$

To determine $k_{p1}(y,y_o)$ and $k_{p2}(y,y_o)$ we will use the expressions given in [1] for the displacements at $z = x+iy$ of a plate with a crack and subjected to concentrated forces $X + iY$ acting at $z_o = x_o + iy_o$.

$$
\begin{aligned}
2\mu_p(u_p + iv_p) = S\{ & -\kappa[\log(z-z_o) + \log(\bar{z}-\overline{z_o})] \\
& + \frac{\kappa}{2}[\theta_1(z,z_o) + \theta_1(\bar{z},\overline{z_o})] - \frac{1}{2}[\theta_1(\bar{z},z_o) + \kappa^2\theta_1(z,\overline{z_o})] \\
& + (\frac{\kappa-1}{2})[\kappa\theta_2(z) - \theta_2(\bar{z})] + (\frac{z_o-\overline{z_o}}{\bar{z}-z_o})\theta_5(z,z_o)\} \\
& + \bar{S}[(\frac{z-z_o}{\bar{z}-\overline{z_o}}) - 1 + \kappa\theta_3(z,z_o) - \theta_3(\bar{z},z_o) - \theta_4(z,z_o) \\
& + \kappa\theta_4(z,\overline{z_o})] + \text{rigid body displacement}
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
\theta_1(z,z_o) &= \log(zz_o - a^2 + \sqrt{z_o^2-a^2}\,\sqrt{z^2-a^2}) \\
\theta_2(z) &= \log(z + \sqrt{z^2-a^2}) \\
\theta_3(z,z_o) &= \frac{(z_o-\overline{z_o})}{2\sqrt{\overline{z}_o^2-a^2}}[1 + f(z,\overline{z_o})] \\
\theta_4(z,z_o) &= \frac{(z-\bar{z})}{2\sqrt{\bar{z}^2-a^2}} f(\bar{z},\overline{z_o}) \\
\theta_5(z,z_o) &= \frac{(z-\bar{z})}{2\sqrt{\bar{z}^2-a^2}}[f(\bar{z},z_o) - J(z_o)]
\end{aligned} \tag{8}
$$

and

$$
\begin{aligned}
I(z) &= \sqrt{z^2-a^2} - z \\
J(z) &= \frac{z}{\sqrt{z^2-a^2}} - 1 \\
f(z,z_o) &= \frac{I(z) - I(z_o)}{z-z_o} \\
f(z_o,z_o) &= J(z_o)
\end{aligned}
$$

$$\left.\frac{f(\overline{z},z_o) - J(z_o)}{\overline{z}-z_o}\right|_{z=\overline{z_o}} = -\frac{a^2}{2(z_o^2-a^2)^{3/2}}$$

$$S = \frac{X+iY}{2\pi h_p(1+\kappa)} \tag{9}$$

From (7)-(9) the displacements due to concentrated forces

$$X + iY = -P_1(y_o) - iP_2(y_o) = -2\pi h_p(1+\kappa)S(y_o) \quad \text{at } z_o$$

and $X + iY = -P_1(y_o) + iP_2(y_o) = -2\pi h_p(1+\kappa)\overline{S(y_o)}$ at $\overline{z_o}$

can be found by superposition. Hence, after some manipulations

$$u_p + iv_p = S(y_o)k_{p1}(y,y_o) + \overline{S(y_o)}k_{p2}(y,y_o) \tag{10}$$

where

$$\begin{aligned}2\mu_p k_{p1}(y,y_o) &= \kappa[\log(z-z_o) + \log(\overline{z}-\overline{z_o})] - \frac{\kappa}{2}[\theta_1(z,z_o) + \theta_1(\overline{z},\overline{z_o})] \\ &+ \frac{1}{2}[\theta_1(\overline{z},z_o) + \kappa^2\theta_1(z,\overline{z_o})] - (\frac{\kappa-1}{2})[\kappa\theta_2(z) - \theta_2(\overline{z})] \\ &+ (\frac{\overline{z_o}-z_o}{\overline{z}-z_o})\,\theta_5(z,z_o) + 2 - \kappa\theta_3(z,\overline{z_o}) + \theta_3(\overline{z},\overline{z_o}) \\ &+ \theta_4(z,\overline{z_o}) - \kappa\theta_4(z,z_o)\end{aligned}$$

$$\begin{aligned}2\mu_p k_{p2}(y,y_o) &= \kappa[\log(z-\overline{z_o}) + \log(\overline{z}-z_o)] - \frac{\kappa}{2}[\theta_1(z,\overline{z_o}) + \theta_1(\overline{z},z_o)] \\ &+ \frac{1}{2}[\theta_1(\overline{z},\overline{z_o}) + \kappa^2\theta_1(z,z_o)] - (\frac{\kappa-1}{2})[\kappa\theta_2(z) - \theta_2(\overline{z})] \\ &+ (\frac{z_o-\overline{z_o}}{\overline{z}-\overline{z_o}})\,\theta_5(z,\overline{z_o}) + 2 - \kappa\theta_3(z,z_o) + \theta_3(\overline{z},z_o) \\ &+ \theta_4(z,z_o) - \kappa\theta_4(z,\overline{z_o})\end{aligned} \tag{11}$$

where

$$z = d + iy$$
$$z_o = d + iy_o \qquad (12)$$

It is easily seen that if $S(y_o)$ is defined per unit length of L rather than a pair of concentrated forces, then $k_{p1}(y,y_o)$ and $k_{p2}(y,y_o)$ defined above represent the kernels of (4).

Also note that $v_s(y)$ and $v_p(y)$ as defined in (2) and (4) vanish on the real axis $(y = 0)$.

Finally, from (1), (2), (4) and (6) we arrive at the following system of Fredholm integral equations of the second kind with kernels having logarithmic singularities.

$$P_1(y) + \int_L [k_1(y,y_o)P_1(y_o) + k_2(y,y_o)P_2(y_o)]dy_o = \gamma_1 q f_1(y)$$
$$P_2(y) + \int_L [k_3(y,y_o)P_1(y_o) + k_4(y,y_o)P_2(y_o)]dy_o = \gamma_1 q f_2(y) ,$$
$$y \text{ on } L \qquad (13)$$

where

$$k_1(y,y_o) = \gamma_3 k_{s1}(y,y_o) - \gamma_2 \mathrm{Re}[2\mu_p k_{p1}(y,y_o) + 2\mu_p k_{p2}(y,y_o)]$$
$$k_2(y,y_o) = \gamma_2 \mathrm{Im}[2\mu_p k_{p1}(y,y_o) - 2\mu_p k_{p2}(y,y_o)]$$
$$k_3(y,y_o) = -\gamma_2 \mathrm{Im}[2\mu_p k_{p1}(y,y_o) + 2\mu_p k_{p2}(y,y_o)]$$
$$k_4(y,y_o) = \gamma_4 k_{s2}(y,y_o) - \gamma_2 \mathrm{Re}[2\mu_p k_{p1}(y,y_o) - 2\mu_p k_{p2}(y,y_o)]$$
$$(14)$$

and

$$\gamma_1 = \frac{d_s \mu_a}{4h_a \mu_p}$$

$$\gamma_2 = \frac{\gamma_1}{\pi(1+\kappa)h_p}$$

$$\gamma_3 = \frac{d_s \mu_a}{6E_s I_s h_a}$$

$$\gamma_4 = \frac{d_s \mu_a}{A_s E_s h_a} \tag{15}$$

THE STRESS INTENSITY FACTORS

The stress intensity factors will be defined as

$$K_1 - iK_2 = \lim_{x\to a} [\sqrt{2(x-a)}][\sigma_y(x,0) - i\tau_{xy}(x,0)] \tag{16}$$

Due to symmetry $K_2 \equiv 0$. K_1 can be obtained as follows [2]:

$$\frac{K_1}{\sqrt{a}} = q + \frac{2}{a_o} \int_L \alpha(y_o)\,dy_o \tag{17}$$

where

$$\alpha(y_o) = \mathrm{Re}\left\{ S(y_o) \left[\frac{a_o + I(z_o)}{a_o - z_o} \left(1 + \frac{\bar{z}_o - z_o}{a_o - z_o} \right) + \left(\frac{\bar{z}_o - z_o}{a_o - z_o} \right) J(z_o) - \kappa \frac{a_o + I(\bar{z}_o)}{a_o - \bar{z}_o} \right] \right\} \tag{18}$$

where

$$a_o = \begin{cases} a & \text{for the right tip} \\ -a & \text{for the left tip} \end{cases}$$

$$z_o = d + iy_o \tag{19}$$

and $S(y_o)$ is given by (6).

NUMERICAL SOLUTION AND DISCUSSION

The solution of (13) can be obtained by any standard method. A simple collocation yields

$$P_1(y_i) + \sum_{j=1}^{N} [k_1(y_i,y_j)P_1(y_j) + k_2(y_i,y_j)P_2(y_j)]\Delta y_j$$
$$= \gamma_1 q f_1(y_i)$$

$$P_2(y_i) + \sum_{j=1}^{N} [k_3(y_i,y_j)P_1(y_j) + k_4(y_i,y_j)P_2(y_j)]\Delta y_j$$
$$= \gamma_1 q f_2(y_i) \quad , \qquad i = 1,\dots,N \tag{20}$$

from which $P_1(y_j)$, $P_2(y_j)$, $j = 1,\dots,N$ are found.

Similarly for the stress intensity factor

$$\frac{K_1}{\sqrt{a}} = q + \frac{2}{a_o} \sum_{j=1}^{N} \alpha(y_j)\Delta y_j \tag{21}$$

To compare the results with those of [1] we will choose $\nu = 0.30$, $E = 10^7$psi (aluminum) and $h_p = 0.09$ in. Also, we will assume that $A_s = 0.165\ a^2$, $E_s = 1.24\times10^7$ psi, $h_a = 0.004$ in. and $\mu_a = 1.65\times10^5$ psi. The effect of bending stiffness will be illustrated below by comparing the results of [1] with the results obtained here for $b = 0$, $d/a = 0.5$ and $d_s/a = 0.2$ and various values of I_s ranging from zero to infinity. From Figure 3 we see that $K_1/q\sqrt{a}$ assumes the value 0.817 and 0.509 for the left and right tips respectively if there is no bending stiffness ($I_s = 0$). These agree with the $K_1/q\sqrt{a}$ values obtained in [1]. As I_s is increased $K_1/q\sqrt{a}$ of the left tip asymptotically decreases from 0.817 to

approximately 0.806 (0.011 difference) and of the right tip increases from 0.509 to approximately 0.521 (0.012 difference). Since location of the stringer in this case is such that its effect is maximum, we can expect that these differences of stress intensity factors for any other position of the stringer will be much less. Hence the conclusion which can easily be drawn from these results is that the bending stiffness of a stringer is rather insignificant.

It should also be noted that the downward or upward trend of the curves of Figure 3 is dependent on the location of the stringer. In particular if the stringer is located in the middle ($d = 0$), $K_I/q\sqrt{a}$ for the right as well as the left tip of the crack becomes 0.6786 assuming that $b = 0$, $d_s/a = 0.2$ and $I_s = a^4$. Another expected result will be given below for $d_s/a = 0.2$, $I_s = a^4$:

right tip	left tip	
0.9997	0.9998	$b/a = 0$, $d/a = 10$
0.9985	0.9985	$b/a = 10$, $d/a = 0.5$

which means that the effect of the stringer diminishes substantially as a result of the increasing length of the debonded portion of the stringer or the distance of the stringer to the mid-point of the crack.

Acknowledgement

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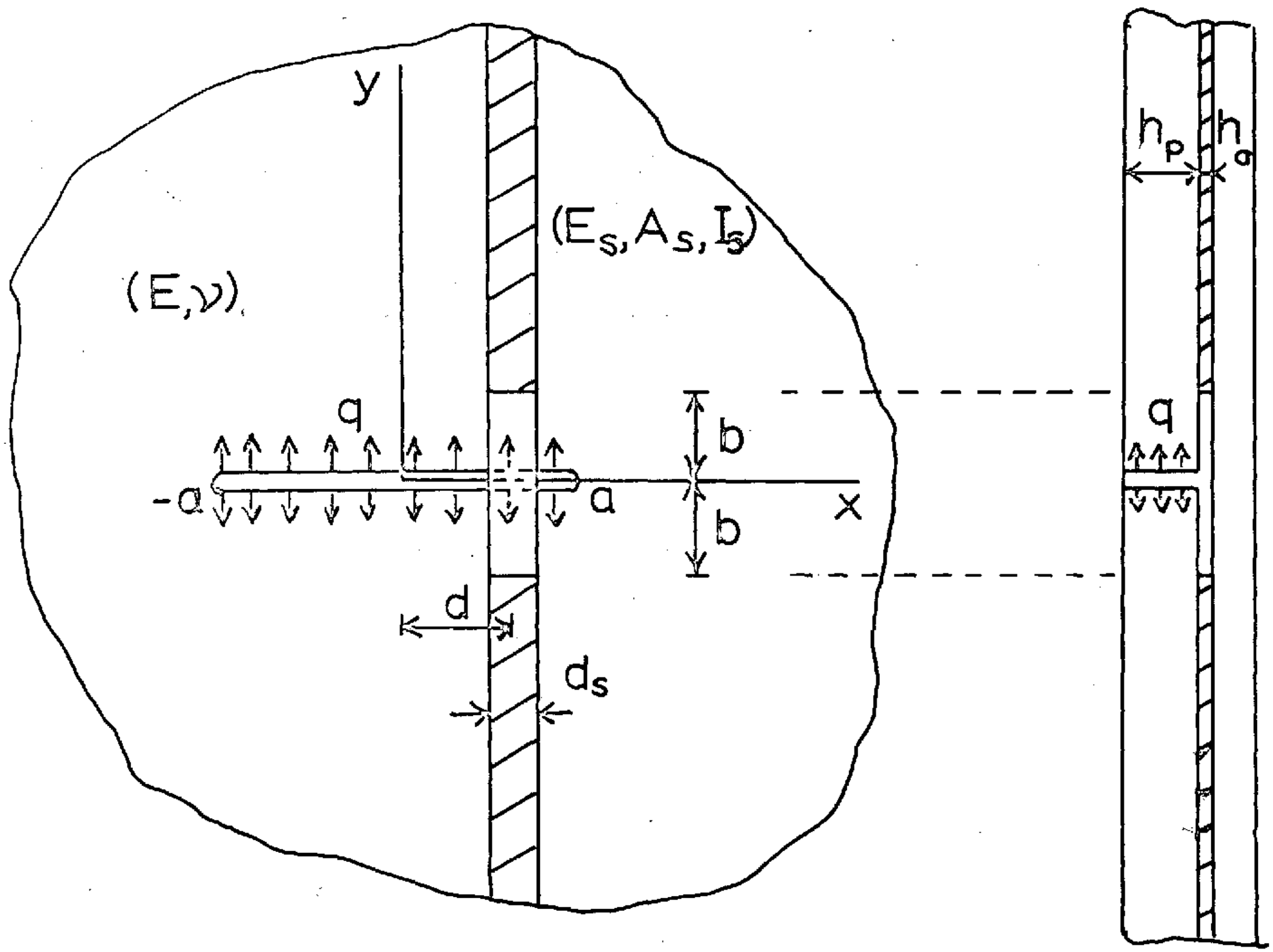

Fig. 1 Geometry of the Problem

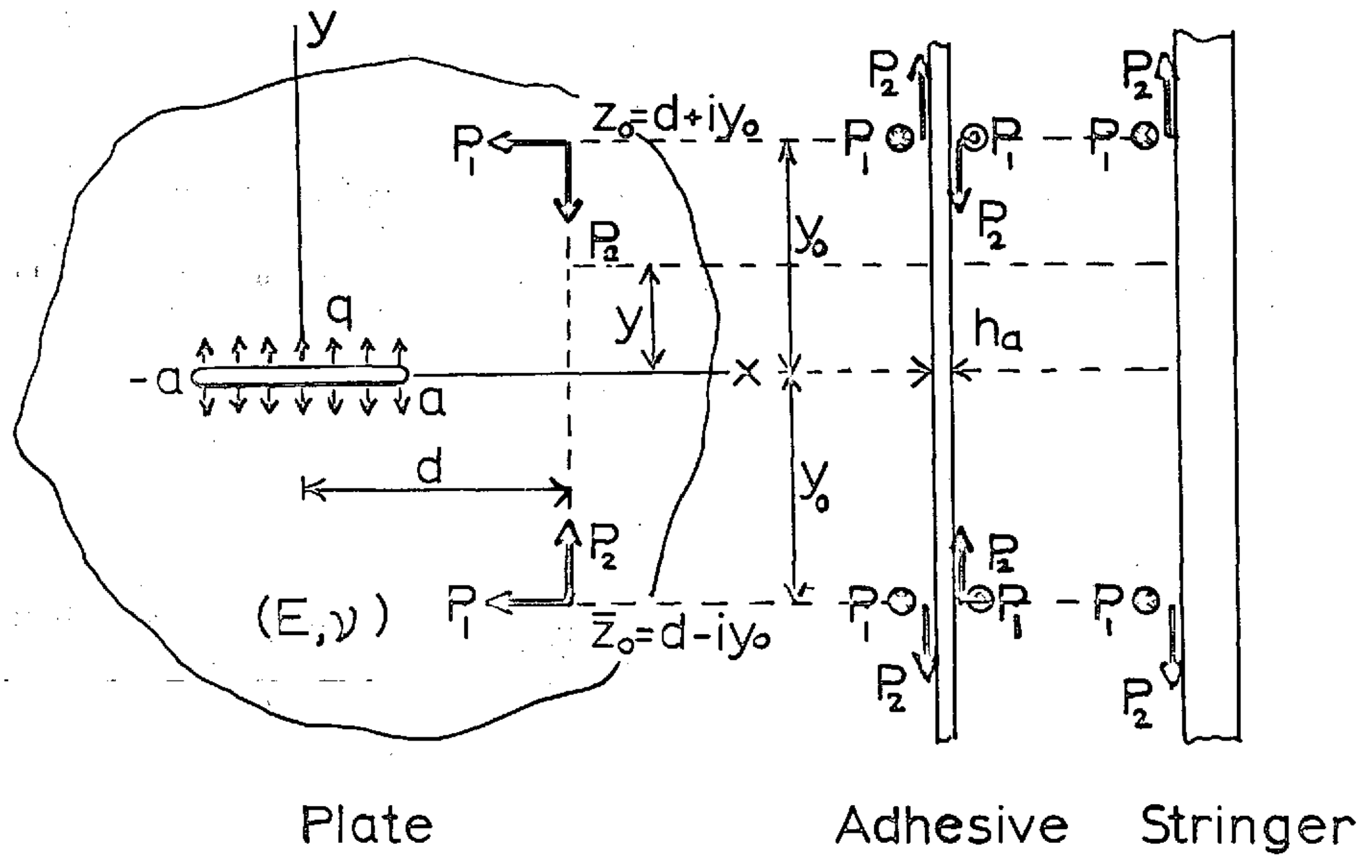

Fig. 2 Free-Body Diagrams of the Plate, the Adhesive and the Stringer

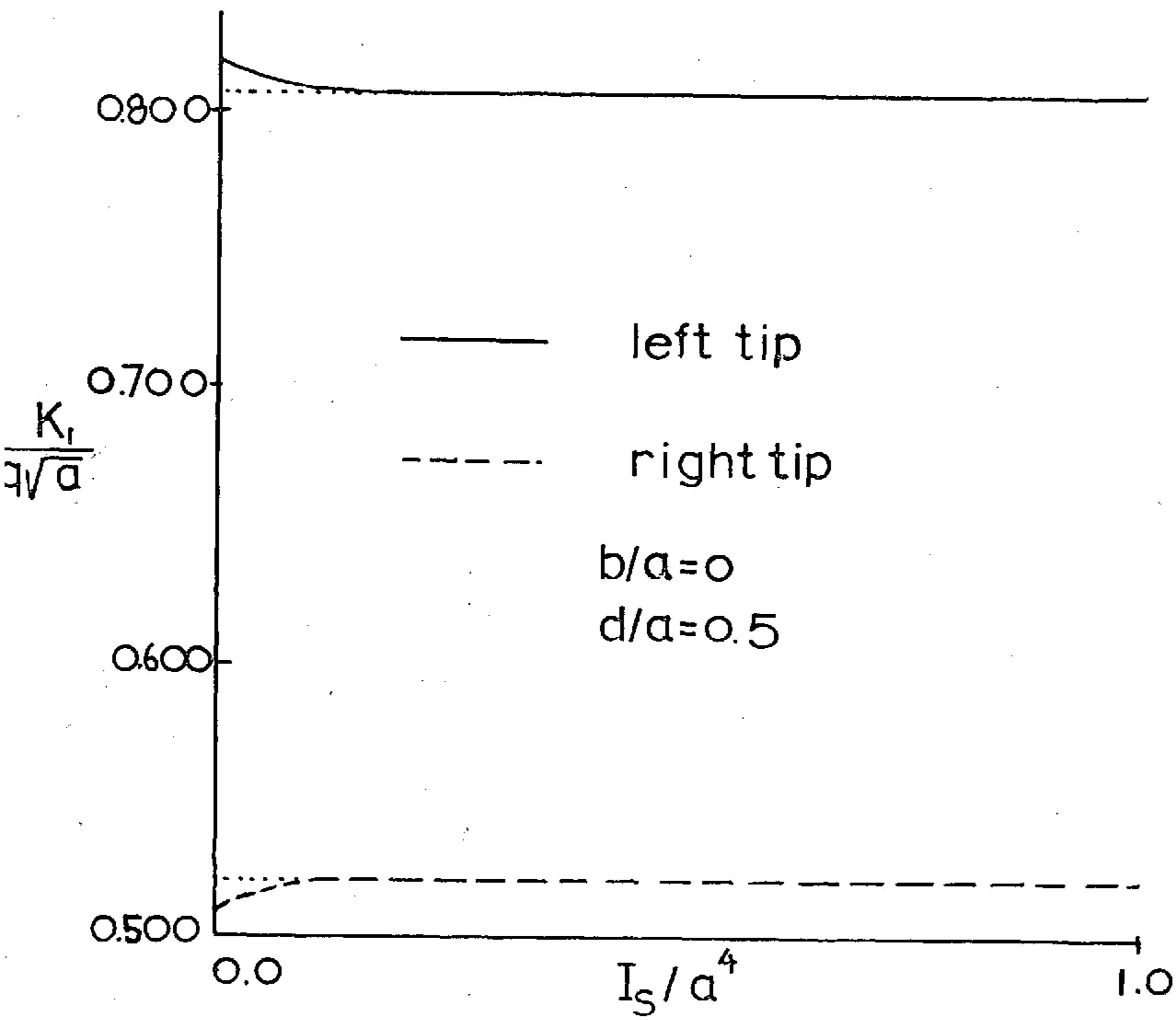

Fig. 3 Effect of Bending Stiffness $\frac{K_1}{q\sqrt{a}}$ vs. I_s/a^4